



# MODELLING FLUVIAL SEDIMENT BUDGETS UNDER UNCERTAINTY

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## 1. INTRODUCTION

The calculation of sediment budgets by differencing a timeseries of digital elevation models (DEMs) has become established as a new methodological paradigm in fluvial geomorphology. This approach has been facilitated by major advances in survey technology, including airborne lidar, digital photogrammetry and rapid kinematic GPS survey. However, despite such developments, the low relief changes typical of fluvial systems imply that errors inherent to survey techniques and surface interpolation methods are likely to propagate significantly into estimates of cut and fill and confound a clear geomorphic interpretation. Recent research has attempted to address this issue by defining a threshold level of detection to distinguish signal from noise in DEMs of Difference derived from a global measure of surface quality. Here we present a framework to extend this approach by developing:

- a theoretical framework for error propagation;
- a spatial model of DEM errors based on a fuzzy inference system;
- a Bayesian approach to incorporating additional data on the spatial context of DEM changes to update local predictions of cut and fill.

## 2. THE STUDY SITE

This new analytical framework is applied to the study of a 1 x 0.3 km reach of the wandering, gravelly River Feshie, Scotland, surveyed using rtkGPS under summer low flow conditions in July 2003 and 2004.



Figure 1. The study area: a) oblique view looking north; b) vertical orthophotograph, July 2000

## 3. DEMs & DEM OF DIFFERENCE

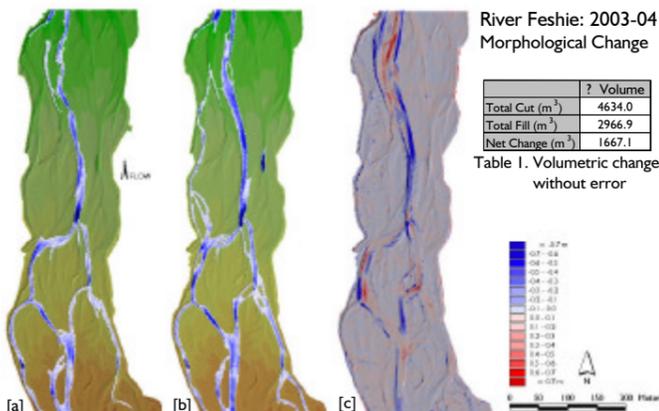


Figure 2. DEMs and DEM of Difference: a) 1m DEM 2003; b) DEM 2004; c) DEM of Difference, DoD (04-03); Table 1. Estimates of cut and fill without error analysis.

## 4. ERROR PROPAGATION

$$[1] z + dz = (x + dx) - (y - dy)$$

$$[2] s_z = \sqrt{s_x^2 + s_y^2}$$

$$[3] t = \frac{[(z_1 - z_2) - 0]}{s_z}$$

where:

$$dx = s_x$$

$$dy = s_y$$

$$s_z = \text{propagated error}$$

$z_1$  and  $z_2$  are geolocated point elevations in two DEMs

If it can be assumed that local DEM errors are random, unbiased and uncorrelated, the errors can reliably be treated as normally distributed, and quantified in terms of their mean ( $\mu$ ) and standard deviation ( $\sigma$ ). For the mathematical operations involving only addition or subtraction the propagation of error into a derived quantity (Eq. 1) then scales as the sum of the errors of the inputs in quadrature (Eq. 2).

Knowledge of the propagated error provides a basis for statistically evaluating the significance of DEM changes by scaling the observed difference relative to the error (Eq. 3) in the form of t-score and testing at any chosen confidence interval.

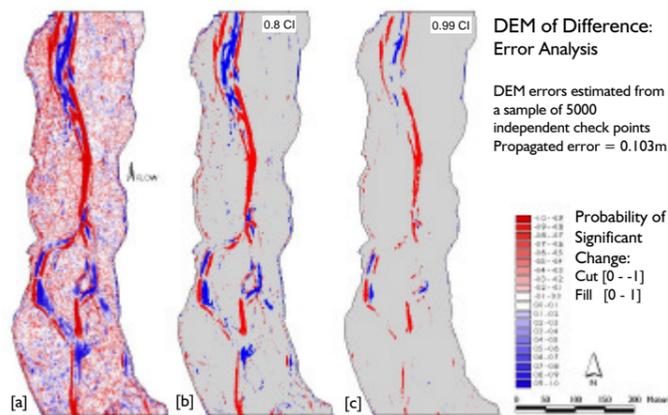


Figure 3. Significance of DEM differences: a) Full scale probability map; b) threshold cut and fill at 0.8 confidence interval threshold; c) 0.99 confidence interval threshold.

## 5. REACH-SCALE BUDGET RELIABILITY

Thresholding predictions of cut and fill enables the sediment budget (ignoring boundary fluxes) to be determined for different confidence intervals. Figure 4 shows a rapid loss of estimated change volumes as the threshold C.I. is increased, although for this analysis, both cut and fill are affected similarly.

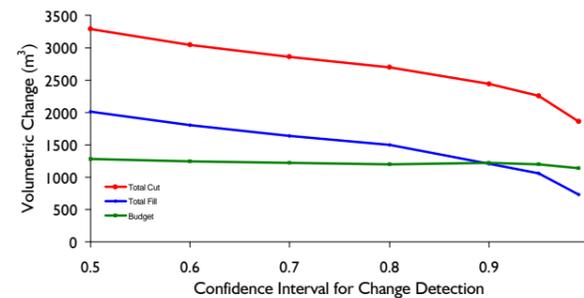


Figure 4. Reach scale volumetric change thresholded at different confidence intervals.

## 6. SPATIAL DEM ERRORS

Estimates of DEM error are typically derived from either measures of instrument precision or through comparison with a sample of observations not used in DEM construction, and the analysis of error conducted presuming global homogeneity of precision. While a practical solution, the interaction of river bed properties with measurement techniques and point density are likely to result in significant spatial variability in quality. For example, Brasington et al. (2000) have shown that the local DEM variance may be correlated with surface roughness and that observations taken while wading are subject to greater uncertainty due to instability. In order to examine the variability of DEM errors, an analysis of 5000 GPS check points surveyed in 2004 were segregated to analyse error properties in: a) bar tops; b) wadable streams; c) slopes greater than 10%. ANOVA revealed significant differences at the 0.0001 significance level (Figure 5 and Table 2).



Figure 5. |Z<sub>obs</sub> - Z<sub>DEM</sub>| revealing bias towards cut banks

	Whole	Bar	Water	Slope
n	4485	2764	844	872
Mean (m)	0.000	0.004	-0.019	-0.012
Mean Abs Error (m)	0.075	0.058	0.072	0.219
SD (m)	0.122	0.081	0.112	0.293
0.9 CI (m)	0.107	0.095	0.085	0.346

Table 2. Check point errors segregated by spatial class

## 7. FUZZY MODELLING

Accounting for spatial variability in vertical DEM uncertainties requires point (cell-by-cell) specification of an error field. While the above analysis shows key factors affecting surface reliability, these are often poorly quantified and vary between survey methods. To reflect this we have developed a generic rule based Fuzzy Inference System to model spatial DEM errors based on: a) theoretical point quality (triangulation error); b) surface roughness (local DEM variance); c) surface slope angle; d) water depth.

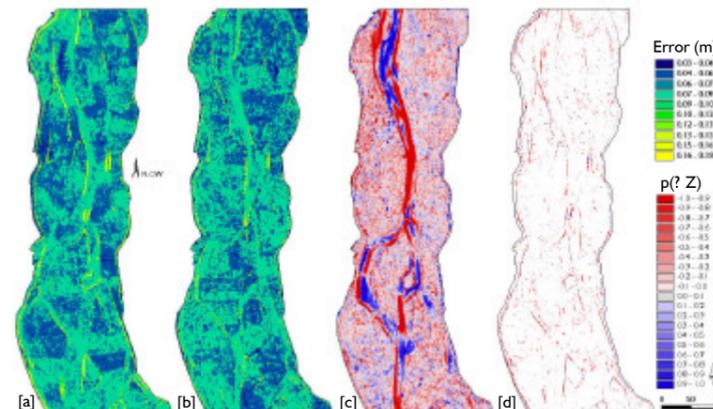
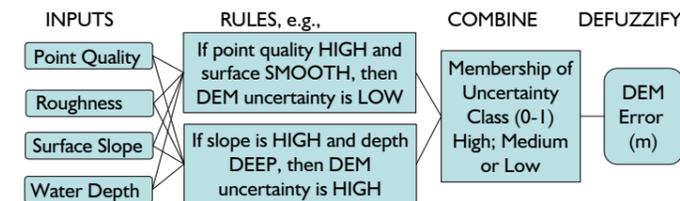


Figure 6. Fuzzy modelling of DEM error: a) fuzzy error 2003; b) fuzzy error 2004; c) significance test (Eq. 3) applied with fuzzy errors; d) comparison with uniform error

## 8. INTERPRETING MORPHOLOGICAL CHANGE

The channel budget reveals little morphological change. Wheaton et al. (2004) classified these changes geomorphically and found that while bank erosion affects only 3% of the reach, it contributes > 20% of the total cut.

Uniform Error	Bar Development	Obscure Changes	Within Class	Bank Erosion	Channel Scour
Total Cut (%)	0.9	3.2	39.5	23.1	33.4
Total Fill (%)	26.1	5.2	58.3	9.2	1.3
Spatial Error	Bar Development	Obscure Changes	Within Class	Bank Erosion	Channel Scour
Total Cut (%)	0.9	2.8	38.7	22.5	35.0
Total Fill (%)	27.3	4.7	57.8	8.8	1.4

Table 3. Channel changes by geomorphological class (after Wheaton et al. (2004))

Differences between the spatial and uniform error models are small due to the quiescent conditions. Previous research however, has shown that error analyses may lead to important differences in interpretation. In particular, the wide but low relief of depositional sheets vs. the deep but localised pattern of channel scour and bank erosion can bias the loss of cut and fill in the calculation of budgets.

## 9. SPATIAL CLASSIFICATION & BAYESIAN UPDATING

Figure 3 reveals important spatial structure in the patterns of cut and fill well below the two CI thresholds. Such information is lost in a simple error analysis due to the low vertical scale of the changes, and may bias the balance of monitored cut and fill as discussed above. Here we develop a spatial classifier to weight the analysis of vertical changes based on the contiguity of such spatial patterns. This classifier analyses the number of cells in a moving 5 x 5 m window that record a similar pattern (cut or fill) to the central cell. This number (0-25) is converted into a spatially-calibrated probability of change (0-1) and used to update the prior probability distribution through Bayes Theorem.

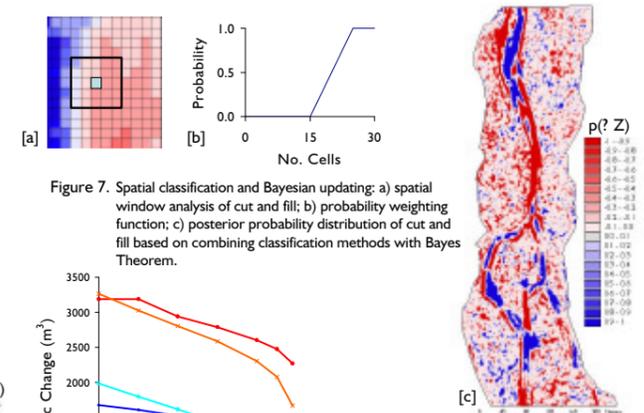


Figure 7. Spatial classification and Bayesian updating: a) spatial window analysis of cut and fill; b) probability weighting function; c) posterior probability distribution of cut and fill based on combining classification methods with Bayes Theorem.

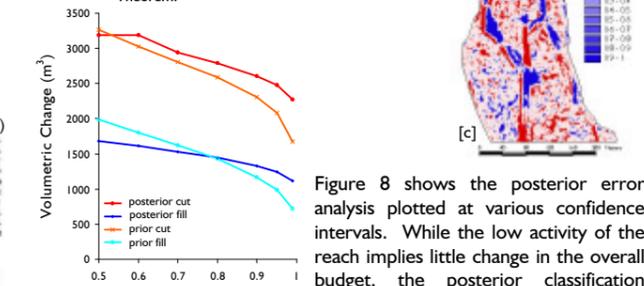


Figure 8. Posterior and prior budget analysis at different CI

## 10. CONCLUSIONS

An automated framework for error analysis of fluvial sediment budgets has been presented. This encapsulates a new approach to modelling DEM errors and a spatial classifier for updating budget predictions using Bayesian statistics.